Penetration of Vertical Jets into Fluidized Beds

An expression has been derived for predicting the penetration depth of vertical jets into fluidized beds based on the experimental observations of several workers. The expression correlates a wide range of data with reasonable accuracy and is suggested as a useful working equation for design purposes.

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SCOPE

The jet phenomenon is of fundamental importance to the performance of fluidized-bed reactors. In the simplest fluidized bed, the fluidizing gas issues from the grid plate in the form of a jet and it is important to know how far such jets will penetrate into the bed; if a high velocity jet impinges on vessel internals, its sandblasting effect is likely to be extremely errosive, with possible disastrous results. For beds in which chemical reactions are taking place, much of the conversion may occur in the jetting region near the distributor plate (Behie and Kehoe, 1973), especially when the reaction is fast.

Despite these facts, the information on jets in fluidized beds available in the literature is sparse. In a previous paper (Merry, 1971), a correlation was developed for predicting the penetration depth of horizontal jets into fluidized beds. This correlation is not directly applicable to the case of a vertical jet, and a separate empirical expression is presented here for predicting the penetration depth of vertical jets into fluidized beds.

CONCLUSIONS AND SIGNIFICANCE

An expression has been developed for predicting the penetration depths of vertical jets into fluidized beds, based on an approximate relationship between the jet length and the initial diameter of the bubble leaving the end of the jet, and an empirical relationship between the jet half-angle and the bed properties. The expression

$$\frac{L}{d_0} = 5.2\left(\frac{\rho_f d_0}{\rho_p d_p}\right)^{0.3}\left[1.3\left(\frac{u_0^2}{g d_0}\right)^{0.2} - 1\right]$$

correlates a wide range of data including gas and liquid fluidized beds, with nozzles of up to 35-mm diameter and jets of up to 0.5 m in length. All the data used are for beds of effectively uniform particle size, and it is not clear at present exactly how the mean particle size d_p in the correlation should be specified for a bed with a very wide range of particle sizes.

A two-dimensional bed was being used to study the flow patterns of particles and fluid in the vicinity of a vertical jet in a fluidized bed. The bed was 0.3 m wide, made of Plexiglas, with 12-mm spacing between front and back walls. The single central vertical jet nozzle was 19 mm wide, and the bed on either side of the nozzle was incipiently fluidized with a fluid supply separate from the jet fluid supply. Experiments were run with a water jet into a water fluidized bed of lead shot; a liquid fluidized bed was used to simplify the task of following the flow of interstitial fluid, and large particles of lead shot (1- and 2-mm diam.) were chosen to give aggregative fluidization. In the course of that work, which will be published shortly, the penetration of the jet was measured and the correlation for predicting the jet penetration depth was developed using this and other data from the literature.

Several workers have presented alternative expressions for jet penetration depth, but generally these are only applicable to the conditions of their experiments. For example, Basov et al. (1969) give the empirical correlation

$$\frac{L}{d_0} = \left[\frac{0.785\, d_p}{0.0007 + 0.566\, d_p}\right] \cdot \frac{u_0^{0.35}}{d_0^{0.3}} \tag{1}$$

where all lengths are expressed in cm. This expression was derived from data obtained in beds of cracking catalyst with d_p between 65 and 540 μm. They predict a slight increase of penetration depth with particle size for given d_0 and u_0. Working with large particles (d_p = 3200 μm), Shakhova (1968) predicts the opposite trend in his expression which can be written

$$\left(\frac{L}{d_0} + 1/2 \cot\theta\right) = 13\left[\frac{\rho_f u_0}{\rho_p \sqrt{g d_p}}\right] \tag{2}$$

where ρ_f and ρ_p are the fluid and particle densities, respectively, and θ is the jet half-angle as shown in Figure 1. And with only four data points, of which two are for 50-μm cracking catalyst and two for 4000-μm wheat seeds, Zenz (1968) presents the expression

$$0.0144\frac{L}{d_0} + 1.3 = 0.5 \log_{10}(\rho_f u_0^2) \tag{3}$$

in which ρ_f is in lb./ft.^{3} and u_0 in ft./s. Equation (3) makes no allowance for particle size.

Markhevka et al. (1971) studied motion pictures of vertical jets at the wall of a cylindrical bed and observe that the bottom region of the jet is conical and that bubbles form in a regular pattern at the end of the jet. The upper ellipsoidal part of the jet, above the conical region, elongates then takes the form of a bubble and separates from the jet. The jet then collapses and the cycle is repeated so that the jet region expands before and during bubble growth and contracts after bubble separation in a rhythmic, pulsating cycle. Figure 1 is a sketch of the jet just before the bubble starts to grow, and of the bubble just after separation, based on their observations. They define the jet penetration depth L as the vertical distance between the orifice and the lower edge of the bubble at the moment of separation. This distance is the maximum length of the jet region and necessarily defines the mean maximum jet length; Markhevka et al. (1971) show that because of the pulsating nature of the jet its maximum length varies by ±20% about this mean. They also observe that the initial diameter D_b of the bubble leaving the end of the jet is approximately equal to the projected diameter of the bottom conical region of the jet at a height L above the nozzle, as shown in Figure 1. Thus D_b can be related empirically to L by the expression

$$D_b = 2\, L_0 \tan \theta \tag{4}$$

where θ is the jet half angle, $L_0 = L + y_0$ is the distance from the apex of the bottom conical section of the jet to the end of the jet, and y_0 $(= 1/2\, d_0 \cot \theta)$ is the distance from the apex to the nozzle. Equation (4) may be rewritten

$$L/d_0 = 1/2 \cot \theta\, (D_b/d_0 - 1) \tag{5}$$

The volume V_0 of a three-dimensional bubble formed at an orifice in an incipiently fluidized bed is given (Davidson and Harrison, 1963, pp. 50-62) as

$$V_0 = 1.138 \left(\frac{G^2}{g}\right)^{0.6} \tag{6}$$

where G is the volume flow rate of fluid out of the orifice. Particle size does affect both wake size and bubble shape (Rowe, 1971), but as a first approximation let us assume that the volume of the bubble wake is one-third the volume of the bubble so that we can express V_0 in terms of D_b. Also expressing G in terms of u_0 and d_0, we can write Equation (6) as

$$(D_b/d_0) = 1.3 \left[\frac{u_0^2}{g d_0}\right]^{0.2} \tag{7}$$

From Equations (5) and (7), we have the form of an expression for jet penetration depth, namely,

$$\frac{L}{d_0} = 1/2 \cot \theta \left[1.3 \left(\frac{u_0^2}{g d_0}\right)^{0.2} - 1\right] \tag{8}$$

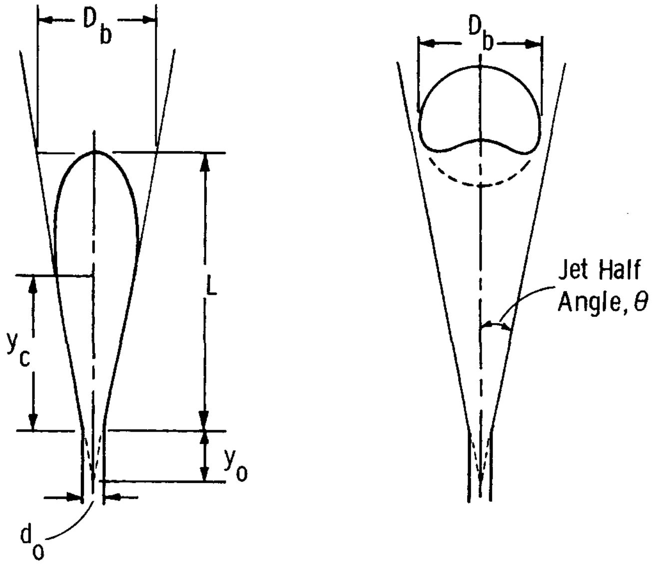

Fig. 1. Geometric arrangement of vertical jet and bubble in a fluidized bed.

Equation (6) relates to bubbles actually forming at an orifice and not at the end of a jet. However, Basov et al. (1969) found an empirical expression for the initial diameter of bubbles leaving their jets,

$$D_b = 0.45\, G^{0.375} \tag{9}$$

where D_b is in cm and G in cc/s, which may be written as

$$(D_b/d_0) = 0.41\, (u_0^{0.375}/d_0^{0.25}) \tag{10}$$

Expressing lengths in centimeters, we find that Equation (7) gives a comparable expression

$$(D_b/d_0) = 0.33\, (u_0^{0.4}/d_0^{0.2}) \tag{11}$$

so that there is justification for using Equation (6) to predict the volume of bubbles formed at the end of a jet.

Markhevka et al. (1971) found that the jet half-angle θ did not change significantly in their experiments. This is not to say that the value of θ is the same for all vertical jets in fluidized beds, but it does allow one to assume that

TABLE 1. PROPERTIES OF FLUIDIZED BEDS FOR WHICH DATA ARE AVAILABLE ON VERTICAL JET PENETRATION INTO INCIPIENTLY FLUIDIZED BEDS. DEDUCED AND MEASURED VALUES OF THE JET HALF-ANGLE

								Jet half angle, degrees		
Source	Bed material	Symbol	d_0 mm	d_p mm	ρ_p kg/m³	ρ_f kg/m³	$\frac{\rho_p d_p}{\rho_f d_0}$	Deduced from Eqn. (8), θ	Measured experimentally	Predicted by Eqn. (12)
Markhevka et al.		■	8				8.34	11.1	15.5	10.4
		△	8	0.14			11.6	9.1		11.4
Basov et al.	Cracking catalyst	▽	14		1,000		8.34	7.9	—	10.4
		□	20				5.84	10.5		9.3
Zenz		▲	19.1	0.05		1.2	2.18	8.9	15	7.0
	Wheat	▼	35	3.8	1,380		115	19.4	—	21.8
Shakhova	Copolymer	●	4	3.2	1,000		666	40.3	35	34.0
Yang and Keairns	Sand	◑	28.6	0.5	2,640		38.5	13.7	—	16.1
	Epoxy shells	◐		2.8	208		17.0	10.3		12.5
This work	Lead	×	19.1	1.0	11,750	1,000	0.615	6.7	7	4.7
	Shot	+		2.0			1.23	8.2	7	5.8

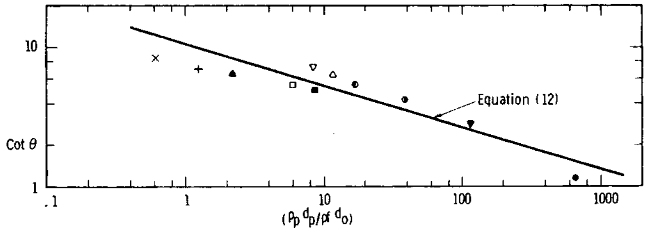

Fig. 2. Logarithmic plot of cot θ v $(\rho_p d_p/\rho_f d_0)$ for data of Table 1.

the value of θ is determined by the properties of the bed. The properties of beds for which data are available on vertical jet penetration are given in Table 1. For each set of data, the mean effective value of θ can be estimated from Equation (8); these calculated values of θ are given in Table 1, alongside a column giving values of the dimensionless group $(\rho_p d_p/\rho_f d_0)$. There does appear to be some relationship between these two sets of numbers, and to establish the form of this, a logarithmic plot of cot θ against $(\rho_p d_p/\rho_f d_0)$ was made, as shown in Figure 2. The result is an empirical expression relating jet half angle θ to bed properties, namely

$$\cot \theta = 10.4 \left[\frac{\rho_p d_p}{\rho_f d_0}\right]^{-0.3} \tag{12}$$

obtained by fitting a straight line to the data.

Experimental measurements of jet half-angle are given for some of the data and these are shown in Table 1 for comparison with the deduced values of θ and the values predicted by Equation (12). It is seen that the measured values of θ compare favorably with the calculated values, giving further justification for the empirical expression above.

Substituting for cot θ from Equation (12) in Equation (8), we obtain the empirical expression for jet penetration depth,

$$\frac{L}{d_0} = 5.2 \left(\frac{\rho_f d_0}{\rho_p d_p}\right)^{0.3} \left[1.3 \left(\frac{u_0^2}{g d_0}\right)^{0.2} - 1\right] \tag{13}$$

All the data from Table 1 are plotted in Figure 3 where it is seen that Equation (13) provides an acceptable correlation, bearing in mind that in practice the maximum jet penetration depth fluctuates $\pm$ 20% about its mean value. Equation (13) is put forward as a working expression for predicting the penetration depth of vertical gas jets into incipiently fluidized beds.

It should be noted that although the correlation has been developed on the basis of a three-dimensional bubble [Equation (7)], the data used includes some obtained in two-dimensional beds. With horizontal jets (Merry, 1971), it was observed that penetration into a two-dimensional bed was approximately twice that into a three-dimensional bed, but the effect appears to be considerably less marked for vertical jets, and it is possible to correlate data from a two-dimensional bed with the expression [Equation (13), Figure 3] which should strictly be applied to a three-dimensional bed.

Two important properties of fluidized beds which have not been included in the correlation are the fluid viscosity and the size range of the particles. The effect of these on jet penetration depth is considered.

FLUID VISCOSITY

The shape of a bubble in an aggregatively fluidized bed is determined by the effective viscosity of the particulate phase (Grace, 1970) rather than by the viscosity of the fluidizing fluid. This effective viscosity of the particulate phase is an order of magnitude greater than the fluid viscosity, even for a water fluidized bed.

The minimum fluidizing velocity of a bed of particles is inversely proportional to the fluid viscosity for small particles, and independent of fluid viscosity for large particles (Kunii and Levenspiel, 1969, p. 73). Equation (6) shows the initial bubble volume to be independent of minimum fluidizing velocity; since this expression has been verified experimentally for beds of small particles (Harrison and Leung, 1961), it can be concluded that the initial bubble volume is also independent of the fluid viscosity. Therefore, it is suggested that the fluid viscosity does not significantly affect the jet penetration depth since it does not affect bubble shape or initial bubble size. The fact that data obtained in a water fluidized bed can be correlated with data from air fluidized beds tends to confirm this conclusion.

A large increase in fluid viscosity will, however, change the situation since it will tend to give particulate rather than aggregative fluidization (Davidson and Harrison, 1963, pp. 80-91). The correlation presented here for jet penetration depth has been developed on the basis of the jet degenerating into a bubble and so is necessarily applicable only to cases of aggregative fluidization.

SIZE RANGE OF PARTICLES IN THE BED

The term d_p in the correlation is taken to be the mean diameter of solid particles in the bed, but it is worth noting that the data presented in Table 1 are for beds of effectively uniform particle size. Kunii and Levenspiel (1969, pp. 67-71) present a technique for estimating the mean diameter of particles in a bed with a distribution of sizes, based on the particle volume to surface-area ratio. This technique is effective for a relatively narrow range of sizes; however, caution should be exercised in applying it to a wide particle size range, for example, the 6 mm-0 coal

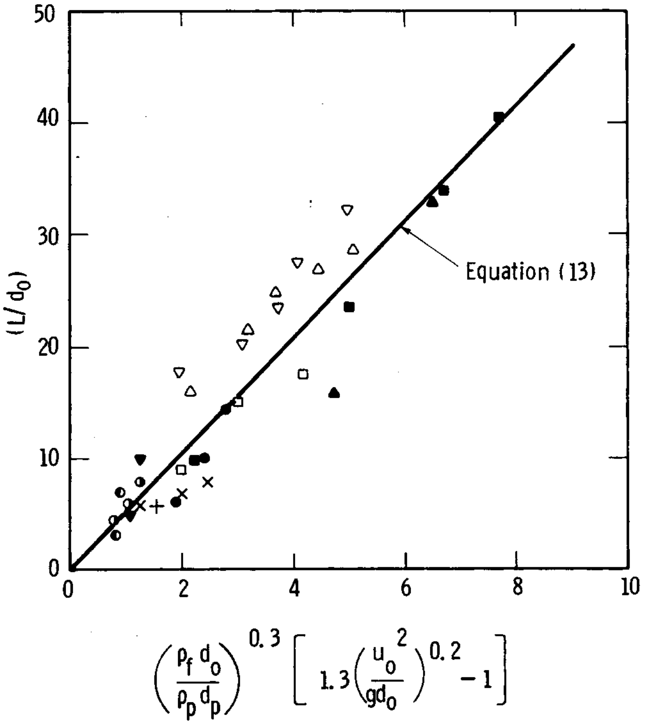

Fig. 3. Correlation of data for penetration depth of vertical jets into fluidized beds.

feed to a typical coal processing fluidized bed (Skinner, 1970); fine particles in the distribution have a considerable effect on the calculation. Similarly, it is not clear at present exactly how d_p should be specified in applying the correlation for jet penetration depth to a bed with a wide range of particle sizes.

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NOTATION

d_0 = jet nozzle diameter
d_p = mean particle diameter
D_b = initial bubble diameter
g = acceleration due to gravity
G = volumetric gas flow rate
L = jet penetration depth measured from nozzle
L_0 = jet penetration depth measured from apex of cone
u_0 = jet nozzle velocity
V_0 = initial bubble volume
y_c = length of conical section of jet ($y_c/L = 0.55$)
y_0 = distance between apex of cone and jet nozzle

Greek Letters

θ = jet half-angle
ρ_f = density of fluid
ρ_p = density of solids

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A Generalized Thermodynamic Correlation Based on Three-Parameter Corresponding States

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The volumetric and thermodynamic functions correlated by Pitzer and co-workers analytically represented with improved accuracy by a modified BWR equation of state. The representation provides a smooth transition between the original tables of Pitzer et al. and more recent extensions to lower temperatures. It is in a form particularly convenient for computer use.

SCOPE

The 3-parameter corresponding states principle as proposed by Pitzer and co-workers has been widely used to correlate the volumetric and thermodynamic properties needed for process design. The original correlations by Pitzer et al., based on that principle, were limited to reduced temperatures above 0.8. Several extensions to lower temperatures have appeared in the last five years. Most of these correlations are in tabular or graphical form, difficult to implement on the computer. Also, significant discrepancies appear at the interface (near $T_r = 0.8$) between the original and extended correlations.

The objective of this work was to develop an analytical correlation, based on the 3-parameter corresponding states principle and covering the whole range of T_r and P_r of practical interest in hydrocarbon processing. Another objective was to improve the accuracy and consistency of the published correlations. This has been achieved by means of two equations of state, similar in form to that of Benedict, Webb, and Rubin, for the simple and reference fluids.